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2002 J. Phys.: Condens. Matter 14 2347

(http://iopscience.iop.org/0953-8984/14/9/323)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **14** (2002) 2347–2352 PII: S0953-8984(02)31192-5

# Application of generalized multifractal analysis for characterization of geological formations

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Received 23 November 2001
Published 22 February 2002
Online at stacks.iop.org/JPhysCM/14/2347

**Abstract**
We demonstrate how texture logs computed from multifractal analysis of dipmeter microresistivity signals can be used for characterizing geological formations (lithofacies) in combination with conventional well logs. In particular, we show that the generalized dimension $D(1)$ (entropy dimension) can be considered as a heterogeneity index providing information on the spatial distribution (amounts of clustering) of heterogeneities in geological sediments. In addition, $D(1)$ logs provide complementary information, compared to the conventional GR-log. This is illustrated by comparing core images of two intervals with similar GR responses and differing $D(1)$ responses. Moreover, the method is equally valid if applied to the texture parameter $D_1(1)$ computed by generalized multifractal analysis. In this way, we propose a tool for extracting textural information from well logging signals, which provides valuable information suitable for integration with data obtained from conventional well logging tools.

## 1. Introduction

Well logging instruments are important tools for the characterization of geological formations in the context of hydrocarbon reservoir exploration. The signals recorded by logging devices reflect physical properties of the rock, such as conductivity, resistivity, acoustical impedance, nuclear properties etc, as a function of depth. For instance, the gamma radiation (GR) log indicates the quantity of clay minerals, which act as gamma ray emitters, present in sandstone as the logging tool passes through the formation [1].

In [2], a method for multifractal analysis of microresistivity well log signals was developed and used to obtain so-called texture logs. They are logs giving multifractal parameters calculated locally in a metre-scale window, versus depth. These texture logs could be used

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to improve our knowledge of geological formations. In particular, the information dimension $D(1)$ could be used as a homogeneity index because it is linked to the spatial homogeneity of the measure, or else its entropy [2]. In [3], the information dimension $D(1)$ was shown to contain statistical information related to the multipoint correlations of the signal from which the generating function is computed. This further supports the choice of $D(1)$ as a texture parameter, describing the texture of the microresistivity signals, and therefore the rock formations.

The proposed methodology was demonstrated [2] using a microresistivity log example from a North Sea well, and the texture log was compared to sedimentary interpretations of core samples from the well. The sedimentary rocks in this case have three major subformations (called lithofacies) denoted $N1$, $N2$ and $N3$. The lithofacies $N1$ are homogeneous and consist of clean sandstone, and they have high $D(1)$ and low GR-log values. The lithofacies $N3$ are heterogeneous and consists of sandstone highly contaminated by shale, and were shown to have low $D(1)$ and high gamma values. Finally, lithofacies $N2$ with only partial contents of shale have intermediate $D(1)$ and GR-log values. For this particular geological example, a clear anti-correlation between the texture log $D(1)$ and GR-log was observed. Lithofacies $N1$, $N2$ and $N3$ originated from channels, crevasses and flood plain deposition environments, respectively. A similar anti-correlation between GR-logs and $D(1)$-logs was observed for five additional wells in [8].

The anti-correlation of the GR-logs and the $D(1)$-logs, and the interpretation of $D(1)$ in terms of lithofacies, demonstrates that the $D(1)$-log contains relevant geological information about the sedimentary rocks. However, in order to demonstrate a practical usefulness of $D(1)$ texture logs, it is essential to find examples of lithofacies where texture log $D(1)$ give complementary knowledge, compared to conventional well logs. A demonstration of such a case is the first goal of this paper.

We wish to emphasize that we are dealing with fractals in nature, and therefore it is important to select correctly the scaling range when computing $D(1)$ [2, 3]. However, the problematic choice of a scaling range can be avoided if one uses a more general approach. The methodology coined 'generalized multifractal analysis' (GMA), which circumvents the problem of a search for an appropriate scaling range, was proposed in [4]. In this method, we choose to describe the generating function for all length scales, not only in the scaling range. The generating function is expanded on a basis of orthogonal polynomials, and the expansion coefficients are the new generalized dimensions. They capture multipoint statistics on all length scales and can be used as alternative indices for texture characterization. For the sake of completeness, this will be explained in the next section.

## 2. Generalization of multifractal analysis

The generating function $\chi_q(\delta)$ used for multifractal analysis is a function of two variables $q$ and $\delta$. $q$ is a real parameter and $\delta \geqslant 0$ is a scale ratio $\delta = \ell/L \leqslant 1$, where $\ell$ is a length scale and $L$ is an upper bound on $\ell$. Multifractal analysis can be performed under the condition that $\chi_q(\delta)$ satisfies the power law property [5]

$$\chi_q\,(\delta) \sim \delta^{\tau(q)} \tag{1}$$

in an interval $\delta\epsilon[\delta_{\min}, \delta_{\max}]$ called *scaling range*. The *mass exponents* $\tau(q)$ [5] are obtained for each $q$ by using linear regressions of $\log(\chi_q(\delta))$ versus $\log(\delta)$ with the model

$$-\ln\lfloor\chi_q(\delta)\rfloor = \tau_0(q) - \tau(q)\ln(\delta). \tag{2}$$

With the change of variable $x = -\ln(\delta) \geqslant 0 \Rightarrow \delta = \exp(-x)$ and the definition $\phi_q \equiv -\ln(\chi_q(\exp(-x)))$, equation (2) can be rewritten in the equivalent form

$$\phi_q(x) = \tau_0(q) + \tau_1(q)\,x. \tag{3}$$

As emphasized in the introduction, a search for a correct scaling range in the linear model (3) can be problematic. If we choose to remain faithful to the idea of using the generating function to characterize texture, then our objective is to find simple and economic descriptions of the function $\phi_q(x)$, whether the measure is multifractal or not, i.e. whether (3) is satisfied or not. Our objective is therefore to generalize the model (3) to handle more general signals, while keeping a formal connection with multifractal analysis.

In [4], it is proposed to generalize the model (3) by expanding the function $\phi_q(x)$ on a family of orthogonal polynomials of increasing order. More precisely, we will write $\phi_q(x)$ in the form

$$\phi_q(x) = \sum_{n=0}^{N} \tau_n(q)\, P_n(x) \tag{4}$$

where $P_n(x)$ denotes a polynomial of order $n$. The model (4) is a straightforward generalization of the simple multifractal model (3). The coefficients $\tau_n(q)$ are formal generalizations of the mass exponent function $\tau(q)$ [5]. In particular, the term $\tau_1(q)$ corresponds to the traditional $\tau(q)$ if the measure is multifractal. We assume in the following that the coordinates $x$ take discrete values $x_i$, $i = 1, 2, \ldots, N$ and that $\phi_q(x)$ is computed only for these values of $x_i$ (we choose equidistant values of $x_i$, as is usually done for multifractal analysis). The polynomials are orthogonal with respect to a scalar product that we define for any two real functions $f$ and $g$ by

$$\langle f, g\rangle = \sum_{i=1}^{N_x} f(x_i)g(x_i) \tag{5}$$

where the summation extends over all values of $x_i$. We construct the polynomials $P_n(x)$ using a Gram–Schmidt orthogonalization (for details, see [4]) and consequently they satisfy the orthogonality relationships

$$\langle P_n, P_m\rangle = \delta_{nm}\langle P_n, P_n\rangle. \tag{6}$$

The coefficients $\tau_n(q)$ in the expansion (4) are simply defined by

$$\tau_n(q) = \langle P_n, \phi_q\rangle / \langle P_n, P_n\rangle \tag{7}$$

which is a consequence of the orthogonality of the polynomials $P_n$. In this representation, the generalized mass exponent $\tau_n(q)$ is simply defined by the projection of $\phi_n(x)$ on the expansion vector $P_n(x)$.

The generating function is defined in such a way that it satisfied the property $\chi_i(\delta) = 1$ for all $\delta$, which is called the *normalization condition* (see definition (11) of [4]). It follows from this condition that $\phi_q(x)$ satisfies $\phi_1(x) = 0$ for all $x$ and therefore $\tau_n(q)$ defined by (7) satisfies $\tau_n(1) = 0$ for each $n$. It follows that the coefficients $\tau_n(q)$ can be always written in the form

$$\tau_n(q) = (q-1)D_n(q) \tag{8}$$

and consequently the polynomial expansion (4) takes the form

$$\phi_q(x) = (q-1)\sum_{n=0}^{N} D_n(q)P_n(x) \tag{9}$$

where $D_n(q)$ are formal extensions of the generalized dimensions [6]. The model (9) should be general enough to describe virtually any measure (the special case $q = 0$ raises some difficulties because $\phi_0(x)$ is not always analytic, and therefore a polynomial fit is not always appropriate). Moreover, the linear component $D_1(q)$ of the fit reduces to the generalized dimension $D(q)$ if the measure is multifractal, i.e. if $N = 1$.

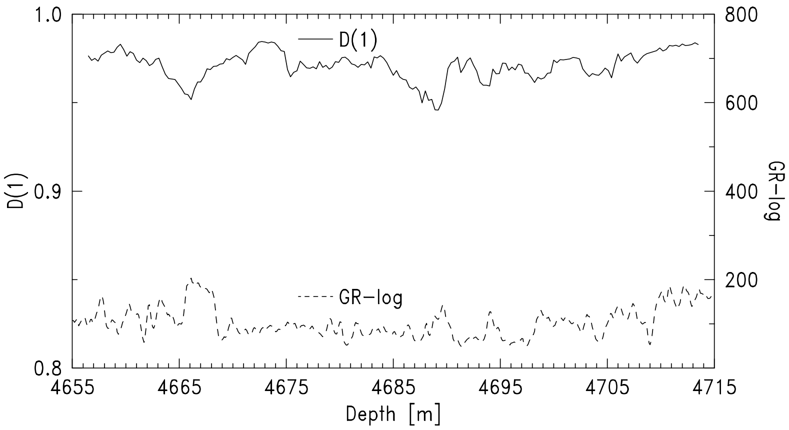


**Figure 1.** Texture log $D(1)$ and GR log from a well (well 1) in the North Sea. The texture log was obtained using the multifractal methods described in [2].

The model (9) allows one to map a function $\phi_q(x)$ of two variables $q$ and $x$ onto several functions $\{D_n(q), n = 0, 1, 2, \ldots, N\}$ of a single variable $q$, which simplifies the representation of $\phi_q(x)$. The number $N$ of functions required to obtain a satisfactory description depends on the accuracy desired. For instance, the chi-square can be used to quantify the quality of fit of the expansion (9) as a function of $N$. As $N$ increases, the accuracy of the expansion improves. By contrast with multifractal analysis, where linear regressions area used to obtain mass exponents, there is no need for any fitting within this new approach. Indeed, the exponents are computed directly from the scalar product with formula (7).

## 3. Results and discussion

The method was applied to a microresistivity log from a North Sea well, that we denote as Well 1. The $D(1)$ texture log was computed from the microresistivity dipmeter log along similar lines as in [2], and the full detail of the analysis is reported in [7]. In figure 1, the GR-log and the $D(1)$ texture log are plotted for an interval 4655–4715 m. If we compare the GR-log and the $D(1)$ log in figure 1, we note a certain anti-correlation, e.g. the two lowest $D(1)$ log intervals correspond to high GR-log values. This observation is in accordance with observations in [2] and [8]. The high GR-log intervals are due to clay minerals, they act as heterogeneities and yield low $D(1)$ values. However, heterogeneity cannot be seen from the GR-log alone. If we focus on the GR-log, it yields high values both in interval A (4665–4668 m) and interval B (4709–4715 m) suggesting lithofacies with high shale content. Based solely on the GR-log, a likely conclusion is to assign the same lithofacies to these two intervals. Using the $D(1)$ texture log, interval A corresponds to low values of $D(1)$, suggesting an uneven (clustered) distribution of heterogeneities (low entropy), whereas for interval B, the opposite is the case. A high value of $D(1)$ in interval B suggests homogeneous sediments with high entropy. In order to verify these findings, we studied the core images of rock samples for these two intervals, displayed in figure 2. The images demonstrate quite clearly that the lithofacies in interval A are different from the lithofacies in interval B. The lithofacies in interval A are heterogeneous with a clustered distribution of shales whereas in interval B, the sediments are homogeneous, however also with a high clay content.

Finally, we computed texture logs using GMA analyses. In figure 3, we display the results for the $D_1(1)$ texture log. We decided to work with the $D_1(1)$ parameter because $D_1(q)$

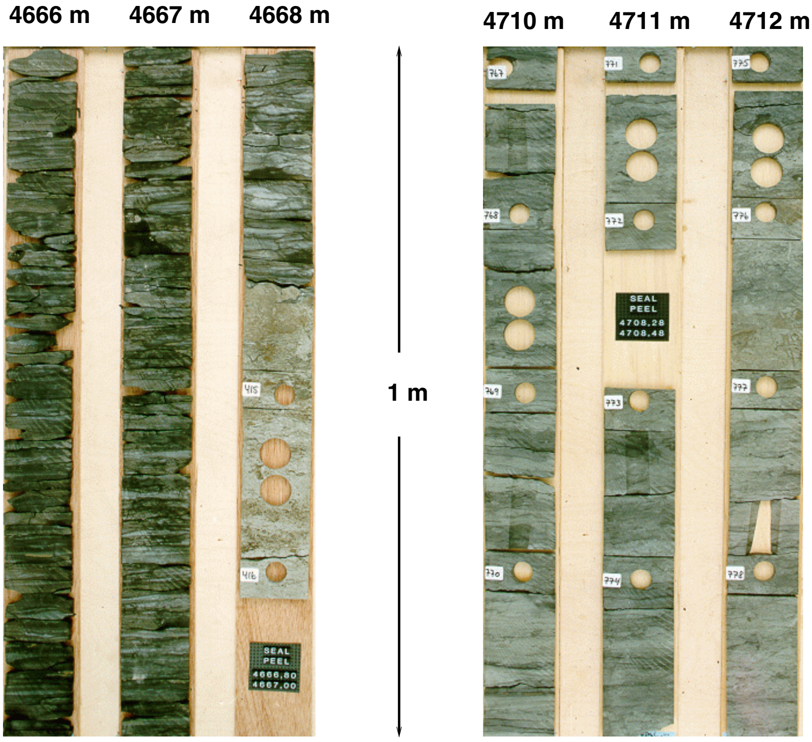


**Figure 2.** Core sample images of two intervals (4666–4668 m) and (4710–4712 m) for the well of figure 1. Although both intervals contain sediments of high clay content and therefore yield high GR-log responses, the textural difference of the two intervals is apparent.

(This figure is in colour only in the electronic version)

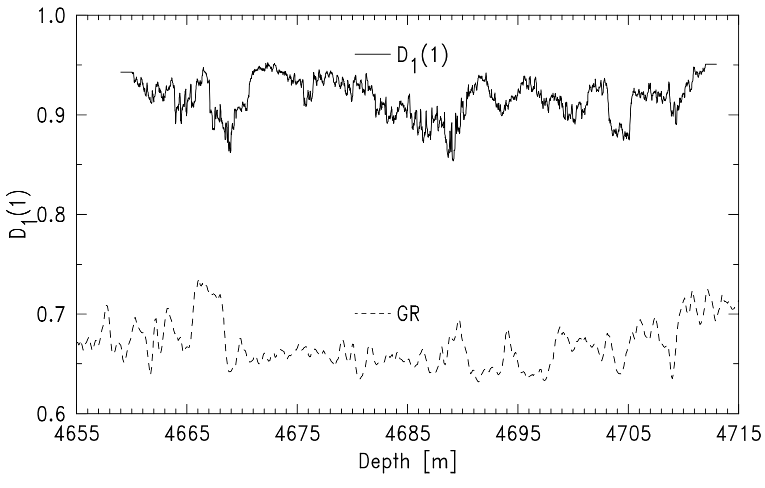


**Figure 3.** Textural log $D_1(1)$ and GR-log from the same well as in figure 1. The $D_1(1)$ log was obtained using the GMA described in [4].

coefficients give the largest contribution to the polynomial expansion (9), and in addition $D_1(1)$ reduces to the generalized dimensions $D(1)$ if the measure is multifractal. The results in figure 3 are comparable with the results in figure 1 suggesting that GMA produces equivalent results to the conventional multifractal analyses in terms characterization of texture logs computed from dipmeter curves.

## 4. Concluding remarks

We have demonstrated how texture logs computed from multifractal analysis of dipmeter microresistivity signals could be useful in characterizing geological formations (lithofacies) in combination with conventional well logs. In particular, we have shown that the generalized dimension $D(1)$ (entropy dimension) could be considered as a heterogeneity index providing information on the spatial distribution (amounts of clustering) of heterogeneities in the form of shales embedded in sandstones. In addition, $D(1)$ logs provide complementary information, compared to the conventional GR-log. This was illustrated by comparing the core images of two intervals with similar GR responses and differing $D(1)$ responses. Moreover, the method is equally valid if applied to the texture parameter $D_1(1)$ computed by GMA, where a choice of a scaling range is avoided by computing the generating function for all length scales. In this way, we are proposing a tool for texture analysis of signals that could be used to integrate microresistivity signals with conventional log responses using standard signal processing tools.

## Acknowledgments

We would like to thank Statoil for providing us with the well log and core data. Valuable discussions with Arild Eldøy are greatly appreciated.

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